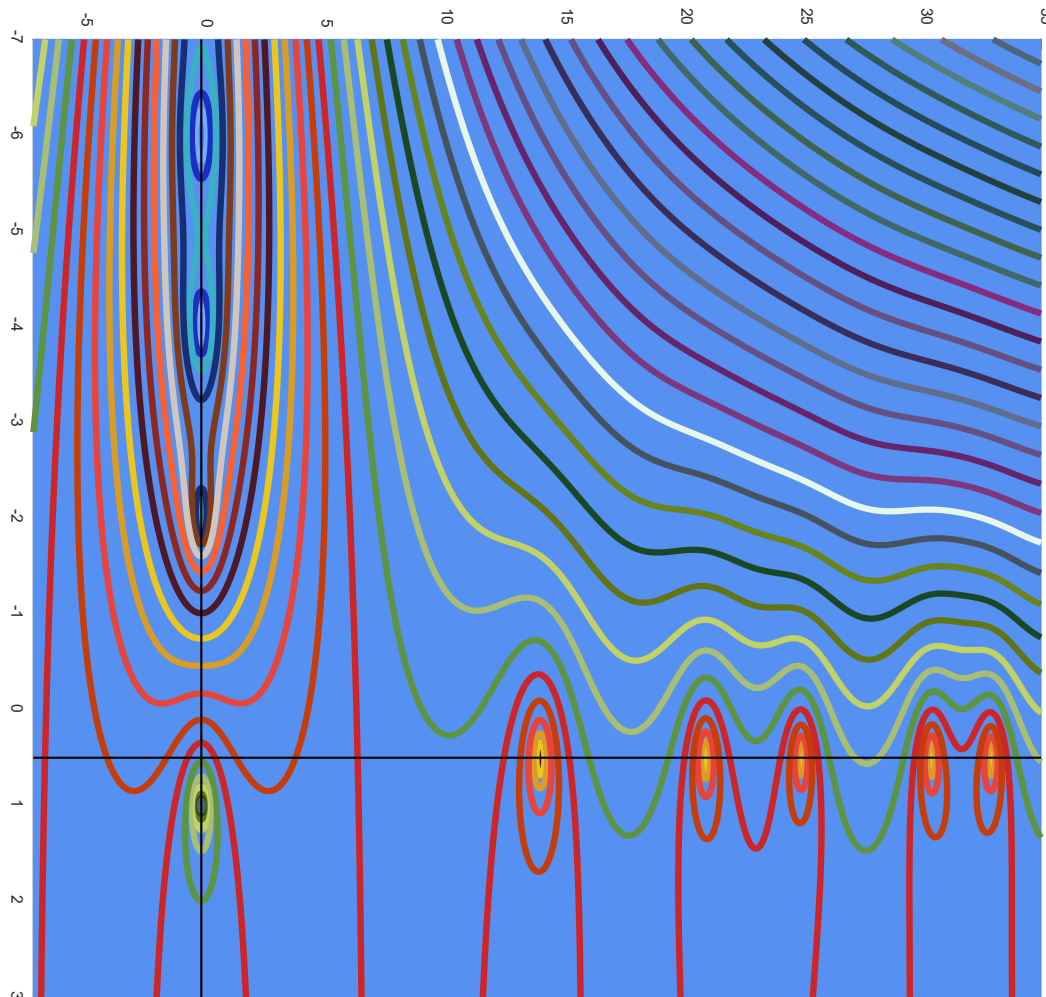# The Riemann hypothesis

The German Georg Friedrich Bernhard Riemann [1826-1866] is one of the greatest mathematicians of all time. He contributed to analysis (see 3-D model of the Riemann surface of the square root function), differential geometry (Riemannian geometry), and to number theory, the purest field of mathematics.

Prime numbers have been puzzling mathematicians for centuries, and they still do. The prime counting function $\pi(x)$ counts the number of primes that are smaller than x. The prime number theorem says that the prime counting function is approximately equal to the offset logarithmic integral function. If the Riemann hypothesis is true, the correction is of order $\sqrt{x}\log(x)$, a far better estimate of the error than is available today.

However, no one knows whether the Riemann hypothesis is true or not.

The Riemann hypothesis asserts that all non-trivial zeros of the Riemann zeta function, defined by (the analytic continuation of)

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$$

are on the critical line, which is where the real part of z is equal to ½.