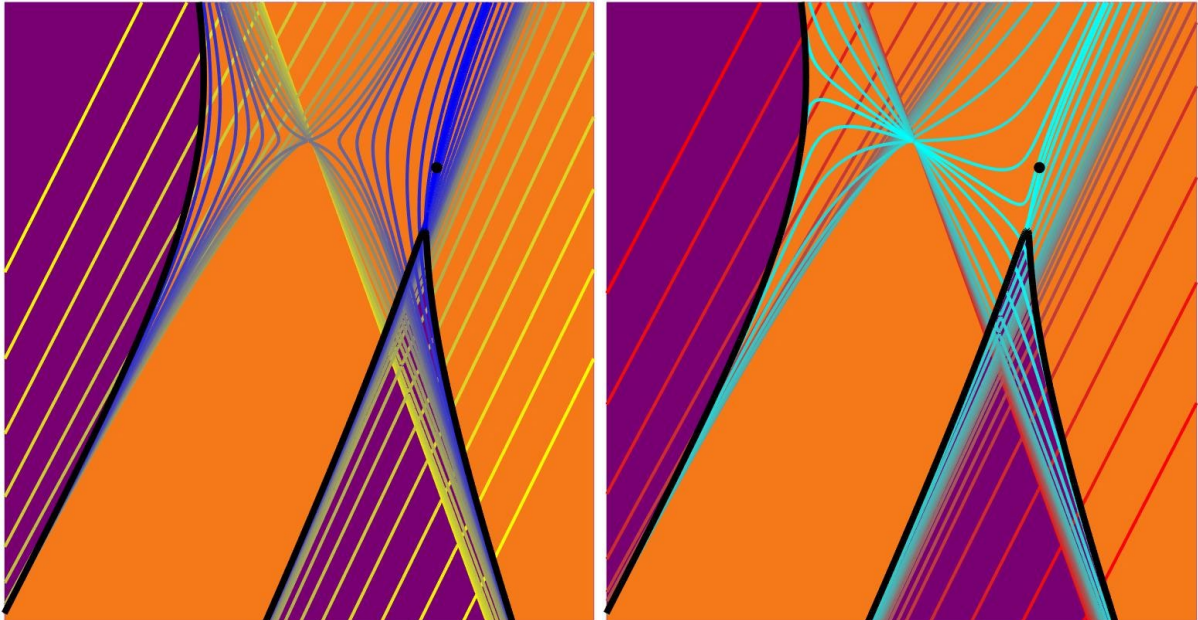# The real tip of complex icebergs

The zero-level set of a function of two real variables is a small part (a 1-D curve in 2-D) of something much bigger, namely a 2-D surface in 4-D. That object is called a Riemann surface. Because we only know life in 3-D, we are unable to see it. But we can dive in the ocean to get a glimpse or two.

The black curve from A degenerate pencil is an example of a zero-level set of a function of two real variables. We seek to understand what lies underneath the isolated singular point at (10,10).

For The real tip of complex icebergs, we have complexified* both variables. In each of the above pictures, the imaginary part of one of the variables is fixed, and the imaginary part of the other variable is disregarded. On the left, the bluest curve has a tiny imaginary part, the yellower the curve the greater the fixed value of the imaginary part.

For the right picture we disregarded the imaginary part of the variable that was fixed on the left, and vice versa. The reddest curve is furthest away from the real (black) curve. It is deep in the ocean. The most cyan curve is very, very close to the real curve, floating just under the water's surface.

*see 3-D model of the Riemann surface of the square root function, where complexification is explained