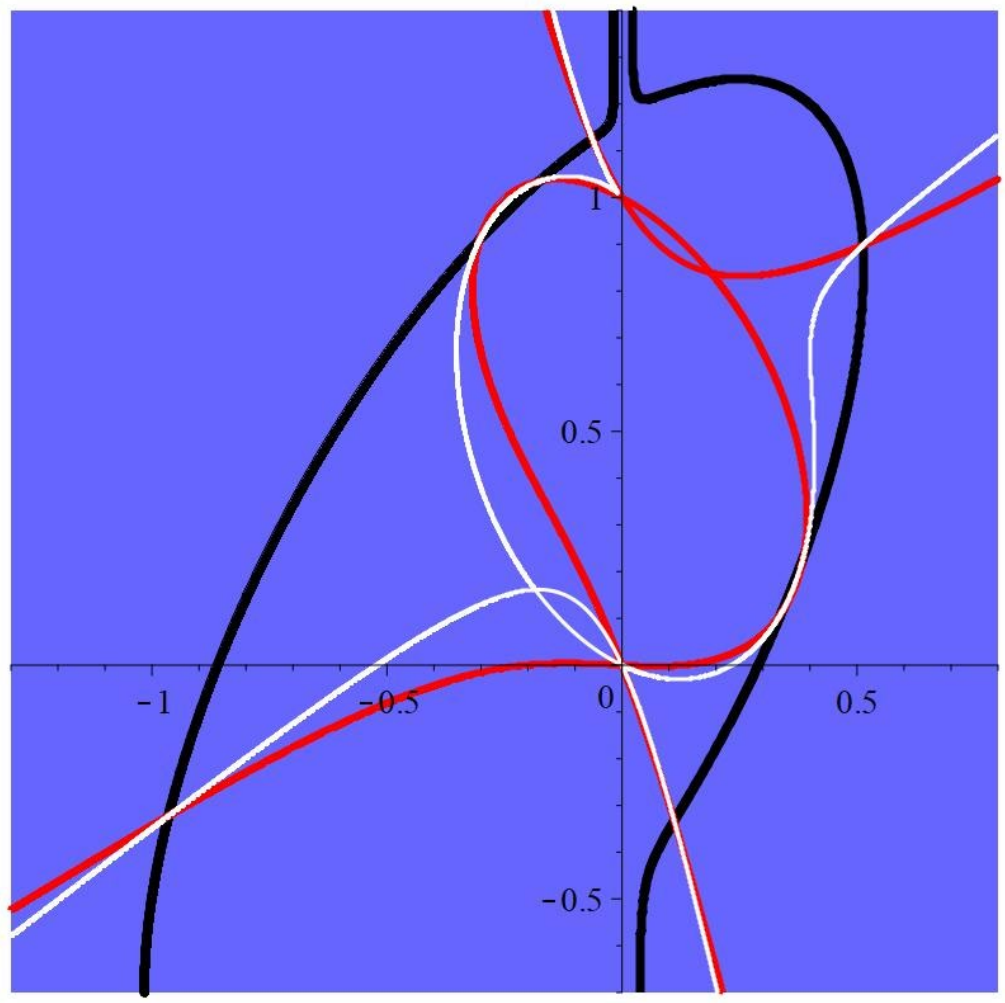
Passing through a crunode



The curves in Approaching an acnode, Passing through a crunode, and On the cusp are quartic curves which all visit the same two points twice, i.e. they come from the same pencil (family) of infinitely many curves with two singular base points at (0,0) and (0,1).

A crunode is another word for an ordinary double point, where a curve goes through the same point twice. Setting aside the two singular base points, there are precisely four curves with one crunode. Two of them are the white and the red curves in Passing through a crunode.

One can imagine the white crunode breaking open, a loop appears and retracts through the origin onto the black curve. One can also imagine the crunode breaking open the other way, and the sharp turn near (0,1) pushing through the second base point forming a loop and meeting itself as the red curve does. When deforming the black curve into the red one must pass through the white crunode.

Passing through a crunode II, III