3-D model of the Riemann surface of the square root function



This 3-D model of the Riemann surface of the square root function is made from copper and solder by Bert van der Kamp. It comprises 17 planar parabolas, as depicted above.

DIY (do it yourself)

To build the 3-D model of the Riemann surface of the square root function, reflect the upper black parabola in the horizontal axis, then place the two black parabolas in parallel planes, so that their minima are closest together, at distance 2.

Next, place the longest parabola (blue) between the end points of the black parabolas, which are furthest apart. If we add direction and number the parabolas, i.e., we say the blue parabola goes from black parabola 1 to black parabola 2, then the next biggest parabola (pink) should go from black parabola 2 to black parabola 1. Keep alternating while adding parabolas until you place the purple parabola in the middle.

The maths behind the model

We start with the relation $0=y-x^2$, which we complexify as $0=y+iz-(x+iq)^2$, where $i^2=-1$, cf. To the i. We expand the bracket and rewrite the complex equation as $0=y-x^2+q^2+i(z-2xq)$. It is equivalent to a system of two real equations $0=y-x^2+q^2$, 0=z-2xq. We fix q and take x as a parameter which runs from -1 to 1. The values of y and z are determined by the above equations. The first equation tells us we are dealing with parabolas, whereas the second equation shows they are planar curves (as the relation between x and z is linear). The 15 fixed values for q are chosen so that the curves are equidistant at x=z=0.